2226 – 5027M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2026

Mathematics: analysis and approaches

Higher level

Paper 3

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite student errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some students may be advantaged for that specific question item, it is likely that these students will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that students may be using an incorrect value.
- If the student's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when students fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a student makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a student incorrectly copies values or information from the question, this is a mis-read (***MR***). A student should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of students' own work does **not** constitute a misread, it is an error.

- If a student uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Students will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the student work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however students may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Students are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

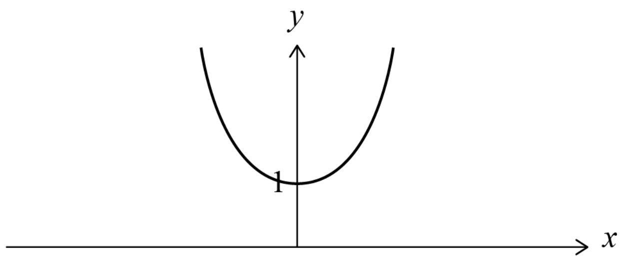
A GDC is required for this paper, but if you see work that suggests a student has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10 Presentation of student work

Crossed out work: If a student has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the student indicates that they would like the work to be marked.

More than one solution: Where a student offers two or more different answers to the same question, an examiner should only mark the first response unless the student indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i)



'parabolic' shape, symmetrical about the y- axis with a minimum point on the y- axis ***A1***

y- intercept at 1 ***A1***

[2 marks]

(ii) **EITHER**

$x^4 + x^2 + 1 > 0$ (for $x \in \mathbb{R}$) (or equivalent in words) ***A1***

OR

the graph is (always) above the x-axis (or equivalent) ***A1***

THEN

hence $f(x) = 0$ has no real roots ***AG***

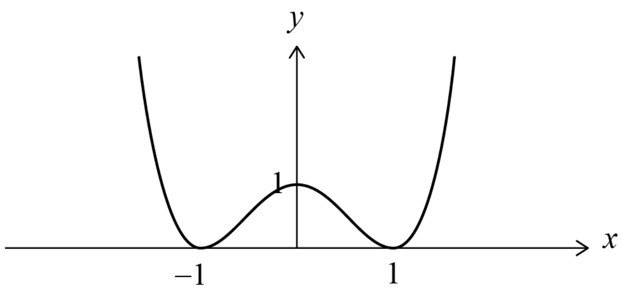
Note:	If given a correct graph in part (a) (i), accept answers such as: 'no x- intercepts'.

[1 mark]

continued…

Question 1 continued

(b)



symmetrical about the y- axis, two minimum points on the x- axis and a maximum point on the y- axis ***A1***

x- intercepts at ± 1 and y- intercept at 1 ***A1***

[2 marks]

(c) (i) $f'(x) = 4x^3 + 2bx$ ***A1A1***

Note: Award at most ***A1A0*** for additional terms seen, for example, '$+d$'.

[2 marks]

(ii) recognizes a condition for an odd function ***(M1)***

$f'(-x) = -f'(x)$ (seen anywhere)

$f'(-x) = 4(-x)^3 + 2b(-x)$ OR $f'(-x) = -4x^3 - 2bx$ ***A1***

$= -(4x^3 + 2bx) = -f'(x)$ ***A1***

so f' is an odd function ***AG***

Note: Award a maximum ***(M1)A0A0*** for solutions that use a specific x- value.

[3 marks]

continued…

Question 1 continued

(d) **EITHER**

at $x = \alpha$, the x- axis is a tangent to the curve ***A1***

OR

the curve touches (does not cross) the x- axis at $x = \alpha$ ***A1***

OR

the graph of f has a turning point on the x- axis at $x = \alpha$ ***A1***

Note: Award ***A1*** for equivalent correct responses provided $x = \alpha$ (position) is stated and either the curve's behaviour or the tangent to the curve at $x = \alpha$ are identified.
The position $x = \alpha$ may be seen in a labelled diagram.

[1 mark]

continued…

Question 1 continued

(e) **METHOD 1**

Note:	The last two marks can be awarded independently of the first two marks.

EITHER

attempts to find $f(-\alpha)$ ***(M1)***

$f(-\alpha)=(-\alpha)^4+b(-\alpha)^2+d$

$=\alpha^4+b\alpha^2+d\left(=f(\alpha)\right)\ (=0)$ ***A1***

since α is a root, then $-\alpha$ is also a root

OR

recognizes that f is even ***(M1)***

$f(-\alpha)=f(\alpha)\ (=0)$ ***A1***

since α is a root, then $-\alpha$ is also a root

THEN

EITHER

uses the result that f' is odd ***(M1)***

$f'(-\alpha)=-f'(\alpha)\ (=0)$ ***A1***

Note:	Award ***A1*** for $f'(-\alpha)=-\left(4\alpha^3+2b\alpha\right)=-f'(\alpha)\ (=0)$.

$-\alpha$ is also a double root of the equation $f(x)=0$ ***AG***

OR

attempts to find $f'\left(-(-\alpha)\right)$ AND $-f'(-\alpha)$ ***(M1)***

$f'\left(-(-\alpha)\right)=-f'(-\alpha)\ (=0)$ ***A1***

Note:	Award ***A1*** for $f'\left(-(-\alpha)\right)=4\alpha^3+2b\alpha=-f'(-\alpha)\ (=0)$.

$-\alpha$ is also a double root of the equation $f(x)=0$ ***AG***

OR

as f is an even function, the graph of f is reflected over the y-axis ***(M1)***

as there is an x-intercept which is a turning point at $x=\alpha$, there will be an x-intercept which is a turning point at $x=-\alpha$ ***A1***

Note:	Award ***(M1)A1*** for showing that $f'(\alpha)=0\Rightarrow f'(-\alpha)=0$.

$-\alpha$ is also a double root of the equation $f(x)=0$ ***AG***

continued…

Question 1 continued

METHOD 2

attempts to show that $f(-\alpha) = f(\alpha) \quad (= 0)$ ***(M1)***

$f(-\alpha) = (-\alpha)^4 + b(-\alpha)^2 + d = \alpha^4 + b\alpha^2 + d = f(\alpha) \quad (= 0)$ ***A1***

recognizes that the sum of roots is zero ***M1***

$\beta - \alpha + \alpha + \alpha = 0$ (where β is the fourth root)

$\beta = -\alpha$ ***A1***

$-\alpha$ is also a double root of the equation $f(x) = 0$ ***AG***

METHOD 3

suppose that β is the other double root

$x^4 + bx + d = (x-\alpha)^2 (x-\beta)^2$ ***(M1)***

attempts to find the coefficient of x^3 OR x in their expansion of $(x-\alpha)^2(x-\beta)^2$ ***M1***

EITHER

coefficient of $x^3 : -2\beta - 2\alpha = 0$ ***A1***

OR

coefficient of x : $-2\alpha^2\beta - 2\alpha\beta^2 = 0$ $(-2\alpha\beta(\alpha + \beta) = 0)$ $(\alpha\beta \neq 0)$ ***A1***

THEN

$\beta = -\alpha$ ***A1***

$-\alpha$ is also a double root of the equation $f(x) = 0$ ***AG***

[4 marks]

(f) (i) **METHOD 1**

attempts to factorize their $f'(\alpha)$ ***(M1)***

$2\alpha\left(2\alpha^2+b\right)(=0)$ OR $\alpha\left(4\alpha^2+2b\right)(=0)$ (or equivalent) ***A1***

($\alpha \neq 0$ as $d \neq 0$)

$2\alpha^2+b=0$ OR $2\alpha^2=-b$ OR $4\alpha^2+2b=0$ OR $4\alpha^2=-2b$ ***A1***

$\alpha^2=-\frac{b}{2}$ ***AG***

METHOD 2 (addressing parts (i) and (ii) simultaneously)

attempts to solve $f(x)=0$ ***(M1)***

$x^2=\frac{-b\pm\sqrt{b^2-4d}}{2}$ ***A1***

as α is a double root, $\sqrt{b^2-4d}=0$ then ***R1***

$\alpha^2=-\frac{b}{2}$ ***AG***

METHOD 3 (addressing parts (i) and (ii) simultaneously)

attempts to complete the square on $f(x)$ ***(M1)***

$f(x)=\left(x^2+\frac{b}{2}\right)^2+d-\frac{b^2}{4}$ ***A1***

as α is a double root and $f(x)$ is expressed as a perfect square ***R1***

$\alpha^2+\frac{b}{2}=0$

$\alpha^2=-\frac{b}{2}$ ***AG***

[3 marks]

continued…

Question 1 continued

(ii) **METHOD 1**

$\left(-\frac{b}{2}\right)^2 + b\left(-\frac{b}{2}\right) + d = 0$ ***(A1)***

$\frac{b^2}{4} - \frac{b^2}{2} + d = 0$ ***A1***

so $d = \frac{b^2}{4}$ ***AG***

METHOD 2

EITHER

the product of roots of $f(x) = 0$ is d ***A1***

OR

$\alpha \times \alpha \times (-\alpha) \times (-\alpha) = d$ OR $\alpha^4 = d$ ***A1***

THEN

($d = \left(\alpha^2\right)^2$ and $\alpha^2 = -\frac{b}{2}$ gives) $d = \left(-\frac{b}{2}\right)^2$ ***A1***

so $d = \frac{b^2}{4}$ ***AG***

Note: Special case if correct **METHOD 2** or correct **METHOD 3** seen in part (f) (i).

METHOD 2 (continued from (f) (i))

$b^2 = 4d$ ***A2***

so $d = \frac{b^2}{4}$ ***AG***

METHOD 3 (continued from (f) (i))

$d - \frac{b^2}{4} = 0$ ***A2***

so $d = \frac{b^2}{4}$ ***AG***

[2 marks]

(g) (i) $b < 0$ ***A1***

[1 mark]

(ii) $b > 0$ ***A1***

[1 mark]

continued…

Question 1 continued

(h) $f''(x) = 12x^2 + 2b$ ***(A1)***

sets their $f''(x) = 0$ ***M1***

$12x^2 + 2b = 0$ OR $6x^2 + b = 0$

for example, $x^2 = -\frac{2b}{12}$ OR $x^2 = -\frac{b}{6}$ OR $x = \pm\frac{\sqrt{-6b}}{6}$ ***A1***

so $x = \pm\sqrt{-\frac{b}{6}}$ ***AG***

Note: Award ***(A1)M1A1*** for any correct expression for x^2 or x in terms of b.

[3 marks]

(i) **METHOD 1**

attempts to find $f\left(\sqrt{-\frac{b}{6}}\right)$ (seen anywhere) ***(M1)***

$$f\left(\sqrt{-\frac{b}{6}}\right) = \left(\sqrt{-\frac{b}{6}}\right)^4 + b\left(\left(\sqrt{-\frac{b}{6}}\right)^2\right) + d$$

$f\left(\sqrt{-\frac{b}{6}}\right) = \frac{1}{36}b^2 - \frac{1}{6}b^2 + d$ OR $f\left(\sqrt{-\frac{b}{6}}\right) = d - \frac{5}{36}b^2$ (seen anywhere) ***A1***

attempts to substitute $x = \sqrt{-\frac{b}{6}}$ and their $f\left(\sqrt{-\frac{b}{6}}\right)$ into $y = -\frac{2\sqrt{6}}{9}(-b)^{\frac{3}{2}}x + c$ ***(M1)***

$\frac{1}{36}b^2 - \frac{1}{6}b^2 + d = -\frac{2\sqrt{6}}{9}(-b)^{\frac{3}{2}}\left(-\frac{b}{6}\right)^{\frac{1}{2}} + c$ OR $d - \frac{5}{36}b^2 = -\frac{2\sqrt{6}}{9}(-b)^{\frac{3}{2}}\left(-\frac{b}{6}\right)^{\frac{1}{2}} + c$ ***A1***

$c = \frac{2}{9}b^2 + \frac{1}{36}b^2 - \frac{1}{6}b^2 + d$ OR $c = \frac{2}{9}b^2 - \frac{5}{36}b^2 + d$ (or equivalent) ***A1***

leading to $c = \frac{b^2}{12} + d$ ***AG***

Note: Award a maximum ***(M1)A0(M1)A0A0*** for a solution that uses $d = \frac{b^2}{4}$.

continued…

Question 1 continued

METHOD 2

attempts to find $f\left(\sqrt{-\frac{b}{6}}\right)$ (seen anywhere) ***(M1)***

$$f\left(\sqrt{-\frac{b}{6}}\right)=\left(\sqrt{-\frac{b}{6}}\right)^4+b\left(\left(\sqrt{-\frac{b}{6}}\right)^2\right)+d$$

$f\left(\sqrt{-\frac{b}{6}}\right)=\frac{1}{36}b^2-\frac{1}{6}b^2+d$ OR $f\left(\sqrt{-\frac{b}{6}}\right)=d-\frac{5}{36}b^2$ (seen anywhere) ***A1***

recognizes that $c=-f'\left(\sqrt{-\frac{b}{6}}\right)\times\left(\sqrt{-\frac{b}{6}}\right)+f\left(\sqrt{-\frac{b}{6}}\right)$ ***(M1)***

Note: Award ***(M1)*** for attempting to substitute their $f\left(\sqrt{-\frac{b}{6}}\right)$ and $f'\left(\sqrt{-\frac{b}{6}}\right)$ into $y-f\left(\sqrt{-\frac{b}{6}}\right)=f'\left(\sqrt{-\frac{b}{6}}\right)\left(x-\sqrt{-\frac{b}{6}}\right)$ (or equivalent).

$c=\frac{2\sqrt{6}}{9}(-b)^{\frac{3}{2}}\left(-\frac{b}{6}\right)^{\frac{1}{2}}+d+\frac{1}{36}b^2-\frac{1}{6}b^2$ **OR** $c=\frac{2\sqrt{6}}{9}(-b)^{\frac{3}{2}}\left(-\frac{b}{6}\right)^{\frac{1}{2}}+d-\frac{5}{36}b^2$ ***A1***

$c=\frac{2}{9}b^2+d+\frac{1}{36}b^2-\frac{1}{6}b^2$ **OR** $c=\frac{2}{9}b^2+d-\frac{5}{36}b^2$ (or equivalent) ***A1***

leading to $c=\frac{b^2}{12}+d$ ***AG***

Note: Award a maximum ***(M1)A0(M1)A0A0*** for a solution that uses $d=\frac{b^2}{4}$.

[5 marks]

Total [30 marks]

2. (a) (i) 4 m *A1*

[1 mark]

(ii) recognizes the initial displacement occurs at $t = 0$ ***(M1)***

$x = 4\sin\frac{\pi}{3}$ OR $x = 4 \times \frac{\sqrt{3}}{2}$ OR $x = 4 \times 0.866025...$

$= 3.46410...$

$= 3.46$ m OR $2\sqrt{3}$ m ***A1***

[2 marks]

(iii) attempts to solve $x = 0$ for t with $t > 0$ ***(M1)***

EITHER

$2t + \frac{\pi}{3} = \pi, (2\pi, ...)$ OR $2t = \pi - \frac{\pi}{3}, \left(2\pi - \frac{\pi}{3}\right), \ldots$ OR $t = \frac{k\pi}{2} - \frac{\pi}{6}$

OR

uses the graph of x to find the first positive t-axis intercept

$t = 1.04719...$

THEN

$= 1.05$ s OR $\frac{\pi}{3}$ s ***A1***

Note:	Award ***(M0)A0*** for $2t + \frac{\pi}{3} = 0$ leading to $t = -\frac{\pi}{6}$ only.

[2 marks]

(b) $\frac{ds}{dt} = nA\cos(nt + b)$ ***A1***

$\frac{d^2s}{dt^2} = -n^2 A\sin(nt + b)$ ***A1***

$= -n^2 s$ ***AG***

Note:	Do not award the final ***A1*** if the ***AG*** line (or equivalent conclusion) is not stated.

[2 marks]

continued…

Question 2 continued

(c) (i) **METHOD 1**

EITHER

$$(a =)\ \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}s}{\mathrm{d}t} \times \frac{\mathrm{d}v}{\mathrm{d}s}$$ ***A1***

OR

$$\frac{\mathrm{d}v}{\mathrm{d}s} = \frac{\mathrm{d}v}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}s} = \frac{a}{v}$$ ***A1***

THEN

$$= v\frac{\mathrm{d}v}{\mathrm{d}s}$$ ***AG***

Note: Award ***A1*** for showing that $v\frac{\mathrm{d}v}{\mathrm{d}s} = a$.

METHOD 2

$$\frac{\mathrm{d}v}{\mathrm{d}s} = -n^2 A \sin(nt+b) \times \frac{\mathrm{d}t}{\mathrm{d}s} \Rightarrow \frac{\mathrm{d}s}{\mathrm{d}t} \times \frac{\mathrm{d}v}{\mathrm{d}s} = -n^2 A \sin\left(nt+b\right)$$ ***A1***

$$v\frac{\mathrm{d}v}{\mathrm{d}s} = a$$ ***AG***

[1 mark]

continued…

Question 2 continued

(ii) attempts to separate variables s and v ***(M1)***

$\int v \, \mathrm{d}v = \int -n^2 s \, \mathrm{d}s$

$\frac{1}{2}v^2 = -\frac{1}{2}n^2 s^2 (+C)$ OR $v^2 = -n^2 s^2 \ (+D)$ ***A1***

EITHER

considers the motion when $t = 0$ ***(M1)***

$s = A\sin b$ and $v = nA\cos b$

$\left(v^2 = -n^2 s^2 + D \Rightarrow\right) D = n^2 A^2 \cos^2 b + n^2 A^2 \sin^2 b$ ***A1***

Note: Award as above for using $s = A\sin\left(nt + b\right)$ and $v = nA\cos\left(nt + b\right)$.

OR

considers the motion where $v = 0 \left(nA\cos(nt + b) = 0\right)$ ***(M1)***

for example, recognizes that $\sin\left(nt + b\right) = 1$

$v = 0$ at $s = \left(\pm\right)A$ ***A1***

OR

considers the motion where $s = 0 \left(A\sin\left(nt + b\right) = 0\right)$ (particle at O) ***(M1)***

for example, recognizes that either $\cos\left(nt + b\right) = 1$ OR $\cos\left(nt + b\right) = -1$

either $v = nA$ at $s = 0$ OR $v = -nA$ at $s = 0$ OR $v = \pm nA$ at $s = 0$ ***A1***

THEN

$C = \frac{1}{2}n^2 A^2$ OR $D = n^2 A^2$

$\frac{1}{2}v^2 = -\frac{1}{2}n^2 s^2 + \frac{1}{2}n^2 A^2$ OR $v^2 = -n^2 s^2 + n^2 A^2$ ***A1***

$v^2 = n^2 \left(A^2 - s^2\right)$ ***AG***

[5 marks]

continued…

Question 2 continued

(iii) **EITHER**

recognizes that the maximum speed occurs where $s = 0$ ***(M1)***

$v^2 = n^2 A^2$

OR

recognizes from $v = nA\cos\left(nt + b\right)$ that the maximum speed occurs when $\left|\cos(nt + b)\right| = 1$ ***(M1)***

OR

recognizes from $v^2 = n^2 A^2 \cos^2(nt + b)$ that the maximum speed occurs when $\cos^2(nt + b) = 1$ ***(M1)***

OR

recognizes from $v^2 = n^2 A^2 \left(1 - \sin^2(nt + b)\right)$ that the maximum speed occurs when $\sin^2(nt + b) = 0$ ***(M1)***

OR

recognizes that the maximum speed occurs where $a = 0$ ***(M1)***

$\sin\left(nt + b\right) = 0 \Rightarrow t = -\frac{b}{n}$

THEN

maximum speed is nA ***A1***

[2 marks]

continued…

Question 2 continued

(d) $\displaystyle\int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{\pi\sqrt{A^2-s^2}}\,\mathrm{d}s \quad \left(=2\int_{0}^{\frac{A}{2}} \frac{1}{\pi\sqrt{A^2-s^2}}\,\mathrm{d}s\right)$ ***(A1)***

$\displaystyle=\left[\frac{1}{\pi}\arcsin\frac{s}{A}\right]_{-\frac{A}{2}}^{\frac{A}{2}} \quad \left(=\left[\frac{2}{\pi}\arcsin\frac{s}{A}\right]_{0}^{\frac{A}{2}}\right)$ ***A1***

attempts to substitute two correct limits into their antiderivative ***(M1)***

$$=\frac{1}{\pi}\left(\arcsin\frac{\frac{A}{2}}{A}-\arcsin\left(-\frac{\frac{A}{2}}{A}\right)\right) \quad \left(=\frac{2}{\pi}\left(\arcsin\frac{\frac{A}{2}}{A}\right)\right)$$

$$=\frac{1}{\pi}\left(\arcsin\frac{1}{2}-\arcsin\left(-\frac{1}{2}\right)\right) \quad \left(=\frac{2}{\pi}\left(\arcsin\frac{1}{2}\right)\right)$$

$\displaystyle=\frac{1}{\pi}\left(\frac{\pi}{6}-\left(-\frac{\pi}{6}\right)\right) \quad \left(=\frac{2}{\pi}\left(\frac{\pi}{6}\right)\right)$ ***A1***

$$=\frac{1}{3}$$

AG

[4 marks]

continued…

Question 2 continued

(e) **METHOD 1**

$\left|v\left(s\right)\right| = n\sqrt{A^2 - s^2}$ (or equivalent) ***(A1)***

$f(s) = \dfrac{k}{n\sqrt{A^2 - s^2}} \Rightarrow \dfrac{1}{\pi\sqrt{A^2 - s^2}} = \dfrac{k}{n\sqrt{A^2 - s^2}}$ ***(A1)***

$\dfrac{1}{\pi} = \dfrac{k}{n}$

$k = \dfrac{n}{\pi}$ ***A1***

METHOD 2

$\left|v\left(s\right)\right| = n\sqrt{A^2 - s^2}$ (or equivalent) ***(A1)***

$\left(f\left(s\right) =\right)\dfrac{1}{\pi\sqrt{A^2 - s^2}} = \dfrac{\frac{n}{\pi}}{n\sqrt{A^2 - s^2}} = \dfrac{k}{n\sqrt{A^2 - s^2}}$ ***(A1)***

$k = \dfrac{n}{\pi}$ ***A1***

METHOD 3

$\left|v\right| = n\sqrt{A^2 - s^2}$ (or equivalent) ***(A1)***

$\left(f\left(s\right) =\right)\dfrac{1}{\pi\sqrt{\frac{v^2}{n^2}}}\left(= \dfrac{1}{\pi\frac{|v|}{|n|}} = \dfrac{|n|}{\pi|v|}\right)$ ***(A1)***

$k = \dfrac{n}{\pi}$ ***A1***

Note: Award as above with use of n.

continued…

Question 2 continued

METHOD 4

$\left|v(s)\right| = n\sqrt{A^2 - s^2}$ (or equivalent) ***(A1)***

$$\int_{-A}^{A} \frac{k}{n\sqrt{A^2 - s^2}}\,\mathrm{d}s = 1 \Rightarrow \frac{k}{n}\left[\arcsin\frac{s}{A}\right]_{-A}^{A} = 1$$

$\frac{k}{n}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 1 \quad \left(\frac{k\pi}{n} = 1\right)$ ***(A1)***

$k = \frac{n}{\pi}$ ***A1***

[3 marks]

continued…

Question 2 continued

(f) (i) **METHOD 1**

$\mathrm{E}(S) = 0$ ***A1***

EITHER

the areas above and below the x-axis are equal ***R1***

OR

the graph is symmetric (has rotational symmetry) about the origin and the upper and lower limits of the definite integral are opposite values, so the definite integral is zero ***R1***

OR

the definite integral of an odd function with limits $\pm A$ is zero ***R1***

Note:	Do not award ***A0R1***.

METHOD 2

$\mathrm{E}(S) = 0$ ***A1***

EITHER

the graph of $y = f(s)$ is symmetrical about the vertical axis ***R1***

OR

f is an even function ***R1***

Note:	Do not award ***A0R1***.

[2 marks]

(ii) **EITHER**

O is the centre of motion ***A1***

OR

O is the mean (expected) position ***A1***

Note:	Award ***A1*** for the answer to part (f) (ii) seen in part (f) (i). Accept answers such as: 'The average position is O'. 'The average displacement from O is zero'. 'The average velocity is zero'. Do not accept answers such as 'simple harmonic motion' without reference made to O.

[1 mark]

Total [25 marks]